Materials discussed on 25/10

We have discussed the following.

1. Show Gauss equation for $\Gamma(z)$:

$$\Gamma(z) = \lim_{n\to\infty} \frac{n^z n!}{z(z+1)...(z+n)}$$

   by using $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt$.

   In the tutorial class, we have shown that

$$\int_0^n (1-\frac{t}{n})^n t^{z-1}\, dt = \frac{n^z n!}{z(z+1)...(z+n)}.$$

   Then by monotone convergence theorem, the limit of L.H.S. is the Gamma function which is analytic.

2. Using Hadamard factorization, show that for any $z \in \mathbb{C}$,

$$\Gamma(z)\Gamma(z+1/2) = 2^{-2z+1}\sqrt{\pi}\Gamma(2z).$$

3. Show the above equation using $B(\alpha, \beta)$. Hints: $B(\alpha,\beta)\Gamma(\alpha+\beta) = \Gamma(\alpha)\Gamma(\beta)$ and try to relate $B(z, 1/2)$ and $B(z, z)$.